Introduction to Engineering Mathematics

With Jim Paradise

Objectives for Today

Our objective for today is not to teach you…

- **Algebra,**
- **Geometry,**
- **Trigonometry, and**
- **Calculus,**

but rather to give you a sound understanding of what each of these are and how, and why, they are used.

My hope is that this will allow you to make informed decisions in the future when choosing math classes.

Definitions

- **Algebra** – the study of mathematical operations and their application to solving equations

- **Geometry** – the study of shapes
  - Algebra is a prerequisite

- **Trigonometry** – the study of triangles and the relationships between the lengths of their sides and the angles between those sides.
  - Algebra and Geometry are prerequisites

- **Calculus** – the mathematical study of change
  - **Differential Calculus** – concerning rates of change and slopes of curves
  - **Integral Calculus** – concerning accumulation of quantities and the areas under curves
  - **Algebra, Geometry, and Trigonometry are prerequisites**

Who needs Calculus?

- **Math Courses Required for B.S. in Engineering Degree**
- Calculus 1 for Engineers
- Calculus 2 for Engineers
- Calculus 3 for Engineers
- Linear Algebra & Differential Equations
-
- **Prerequisite Math Courses for Calculus 1**
- College Algebra **and** College Trigonometry **or**
- Pre-Calculus
-
- **Partial List of Degrees requiring math through Calculus 1 or higher**
- Chemistry
- Geology
- Economics
- Masters in Business Administration
- Math
- Physiology
- Engineering
- Physics

How Old is this stuff?

- **Algebra** – Ancient Babylonians and Egyptians were using algebra by 1,800 B.C.
- **Geometry** – Egypt, China, and India by 300 B.C.
- **Trigonometry** – by 200 B.C.
- **Calculus and Differential Equations** - by the 1,600's

Algebra Properties

Commutative Property

- $a + b = b + a$
- $ab = ba$

Associative Property

- $(a + b) + c = a + (b + c)$
- $(ab)c = a(bc)$

Distributive Property

- $a\,(b + c) = ab + ac$

Rules of signs

- Negative (-) can go anywhere.
- Two negatives = positive

Order of Operations

- **PEMDAS** (Please Excuse My Dear Aunt Sally)
  - Parenthesis and Exponents first, then
  - Multiply and Divide, then
  - Add and Subtract

Exponents and Polynomials

Exponents

- x^2 = x times x
- x^3 = x times x times x times

Polynomials

- $x^2 + 4x + 3$
- $7x^3 - 5x^2 + 12x - 7$

Factoring

- $x^2 + 4x + 3 = (x + 1)(x + 3)$

Solving Equations – Keep Balance

$$3x + 3 = 2x + 6$$

Try to get to form: x = value

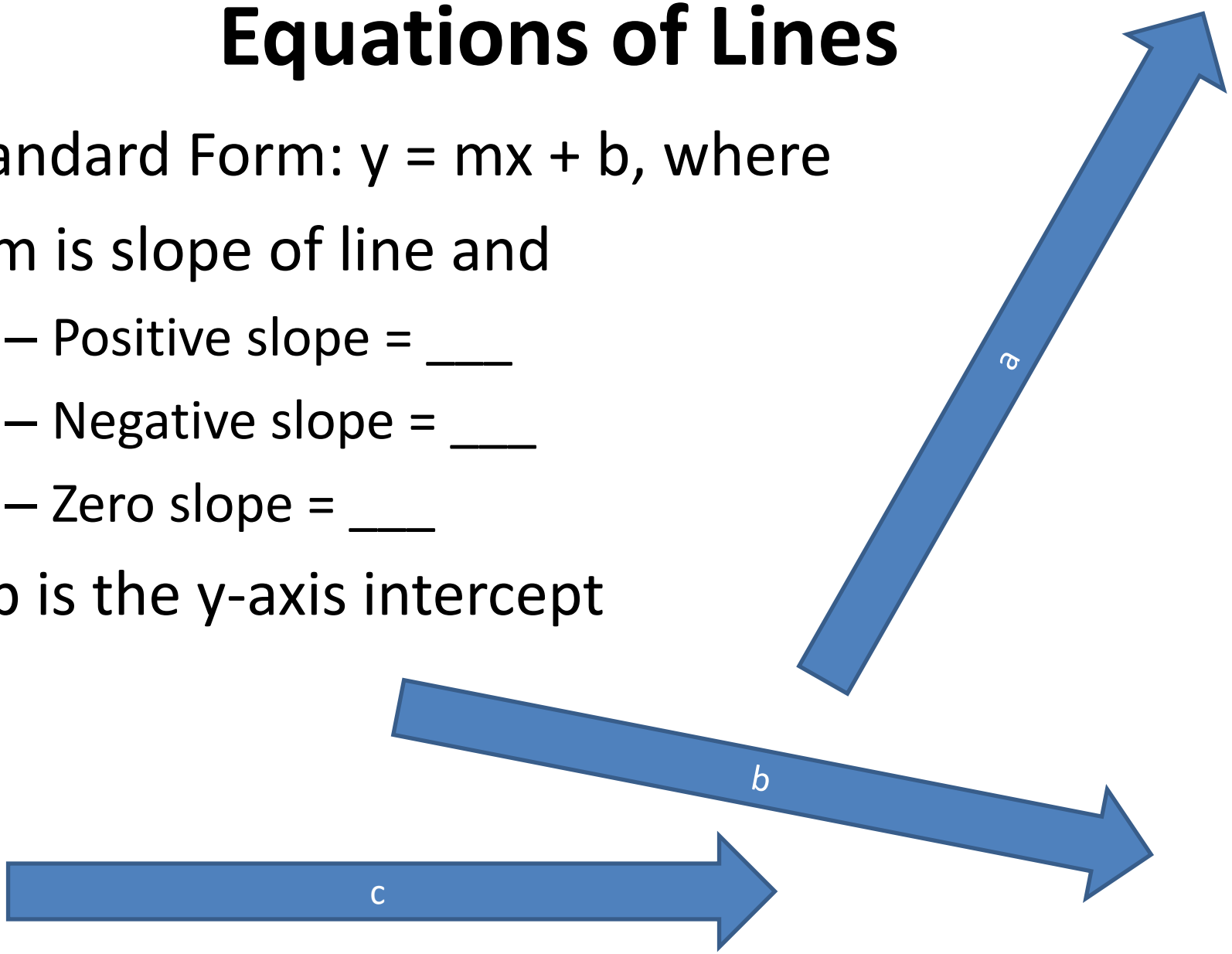
Solving Equations

- **$3x + 3 = 2x + 6$ solve for x**

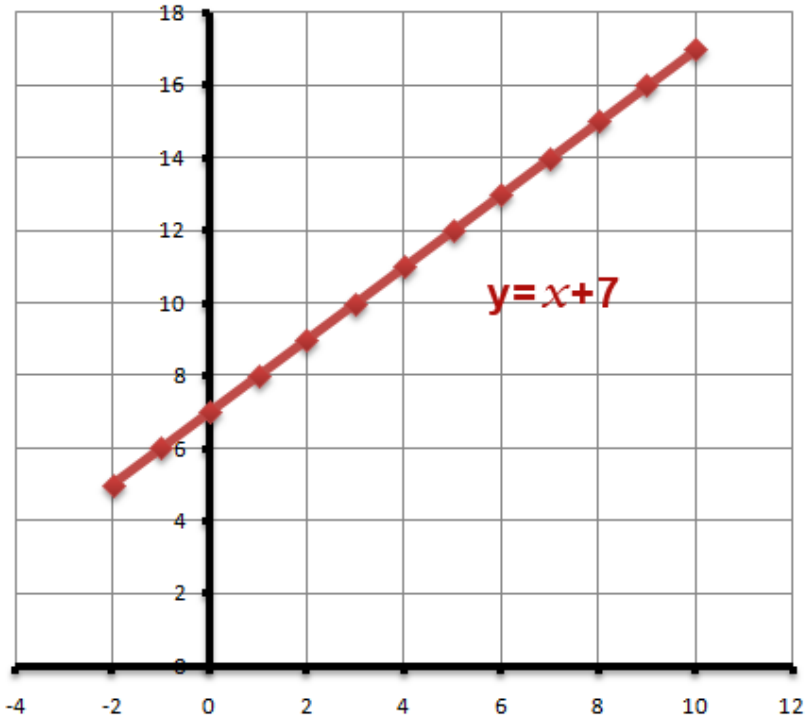
Equations of Lines

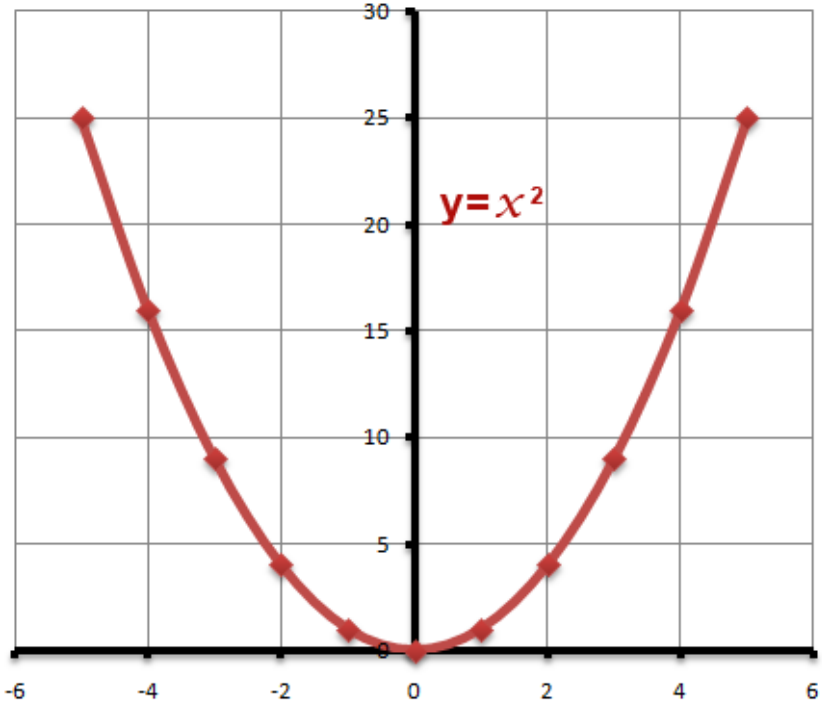
Standard Form: y = mx + b, where

- m is slope of line and
  - Positive slope = ___
  - Negative slope = ___
  - Zero slope = ___
- b is the y-axis intercept

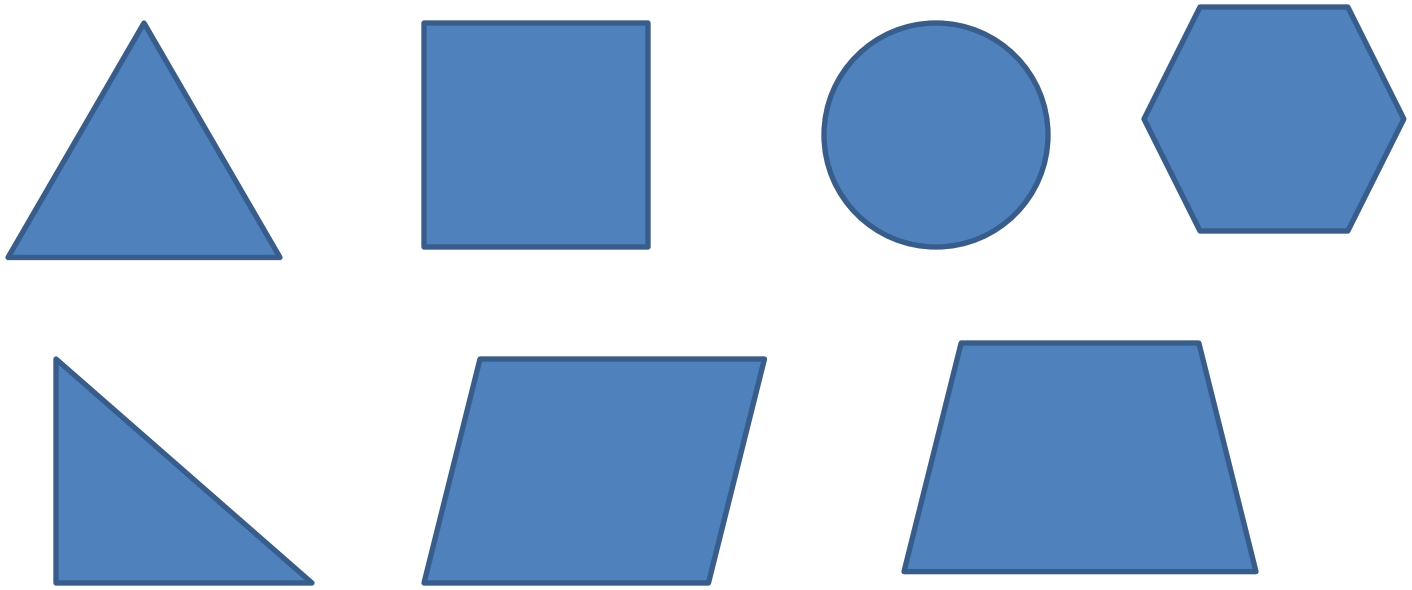


Graphing (2 dimensional)



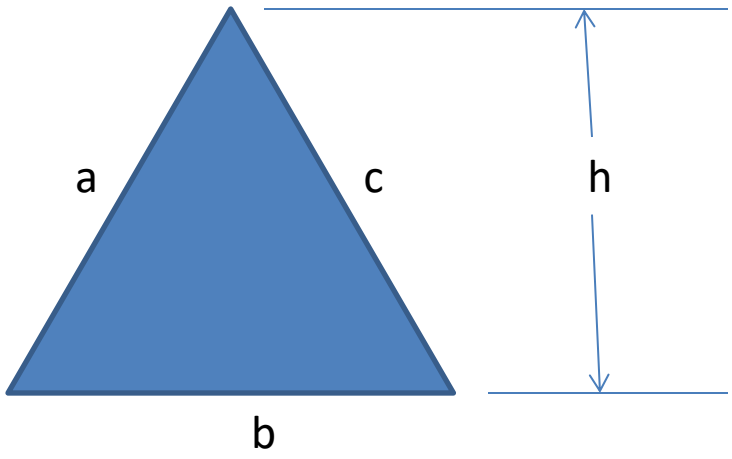


Geometry – the study of shapes



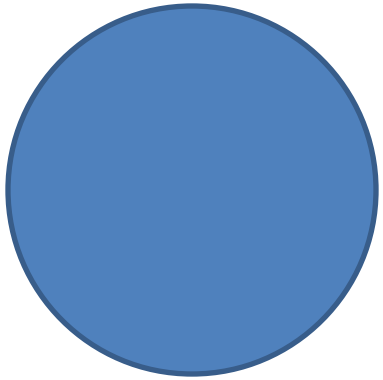
Triangles

- Area = ½ bh where b is base and h is height
- Perimeter = a + b + c
- Angles add up to 180^{o}



Circles

- Area = πr^2 where r is the radius of the circle
- Circumference = $2\pi r = 2d$
- d (diameter) = 2r (radius)



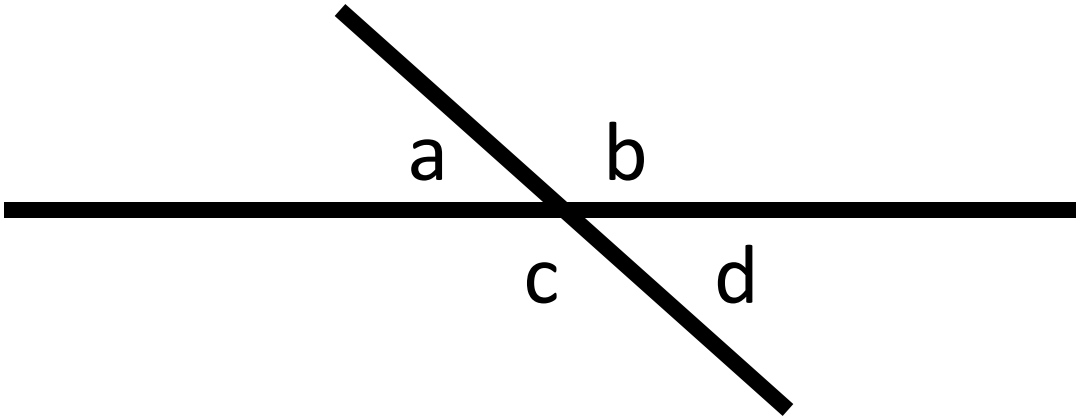
Angles Geometry

Opposite angles are equal

- angle a = angle d
- angle b = angle c

Supplementary angles = 180^o

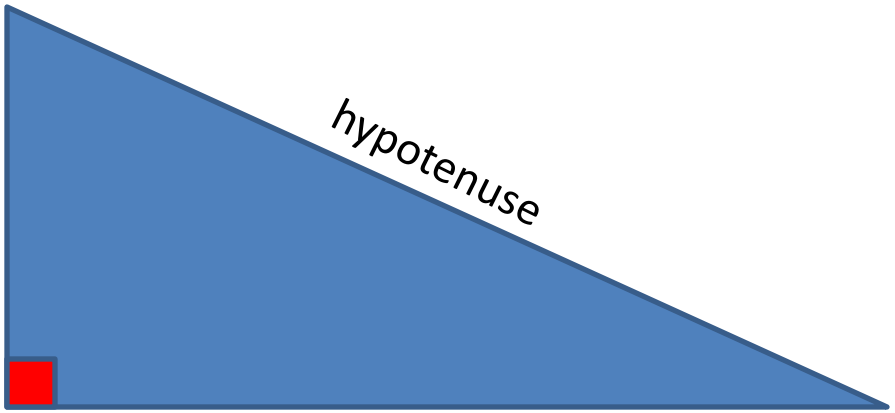
- $a + b = 180^o$
- $b + d = 180^o$
- $c + d = 180^o$
- $a + c = 180^o$



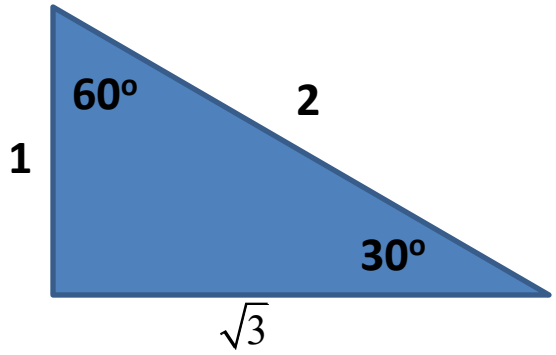
Trigonometry – Study of Triangles

Every **Right Triangle** has three sides

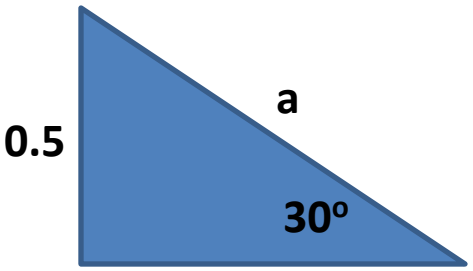
- Hypotenuse
- Opposite
- Adjacent

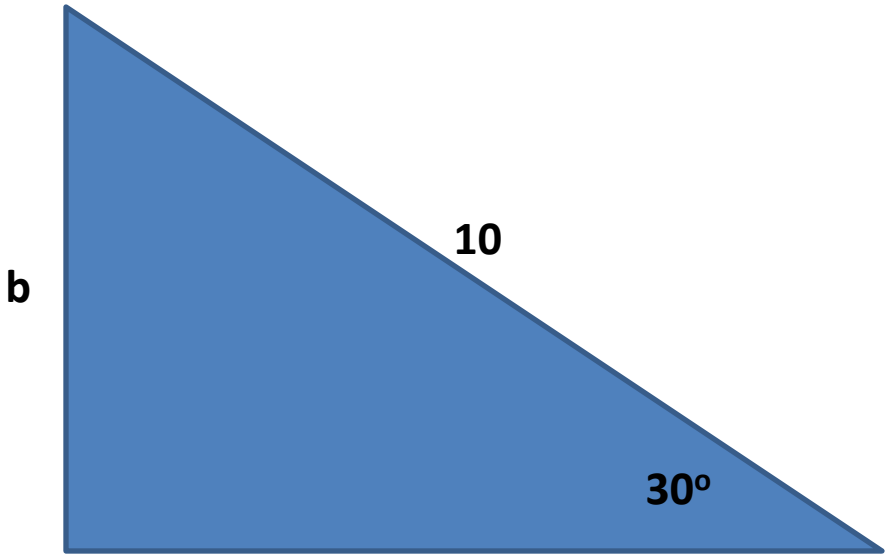


Known Triangle

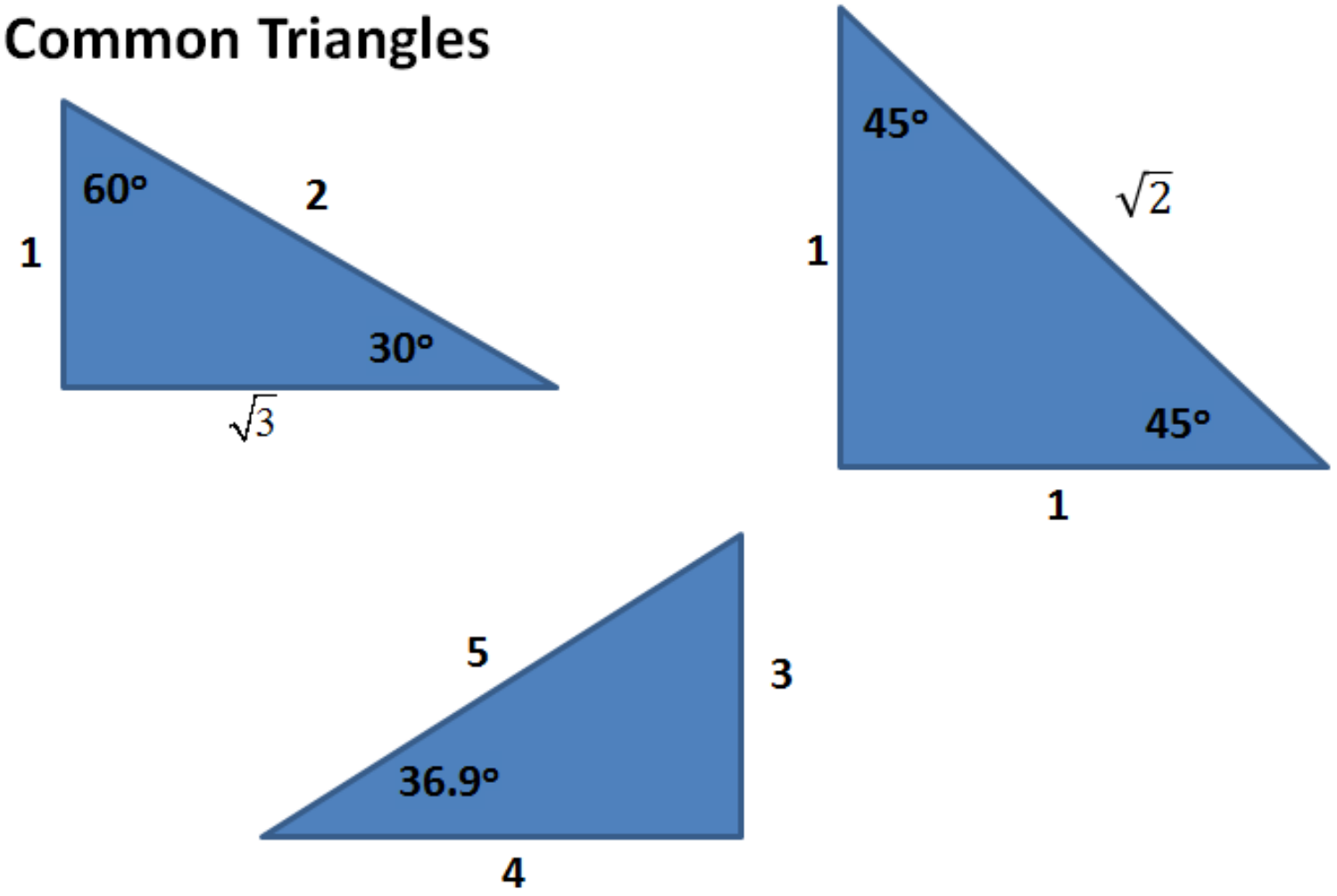


Similar Triangles





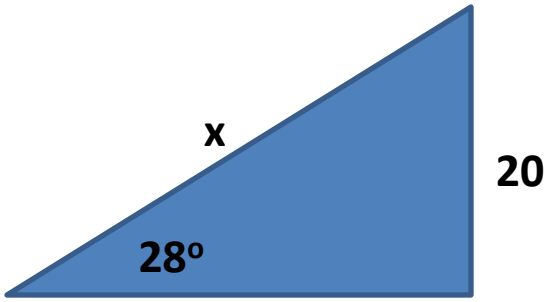
Common triangles

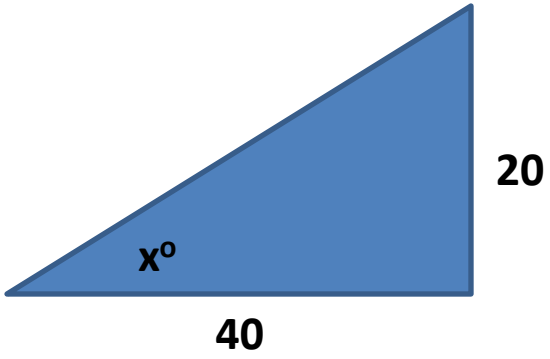


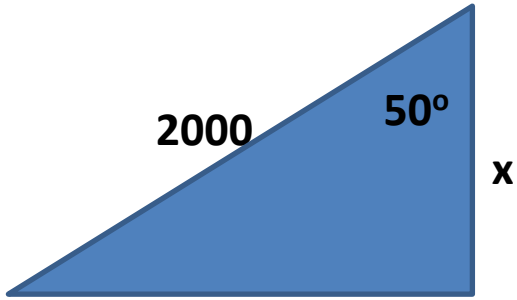
Trig Functions (ratios of triangle sides)

Trig Function		
Name	**Notation**	**Definition**
Sine	**Sin**	$\frac{\text{opposite}}{\text{hypotenuse}}$

Inverse Trig Function		
Name	**Notation**	**Definition**
Sin^{-1}	**Csc**	$\frac{\text{hypotenuse}}{\text{opposite}}$

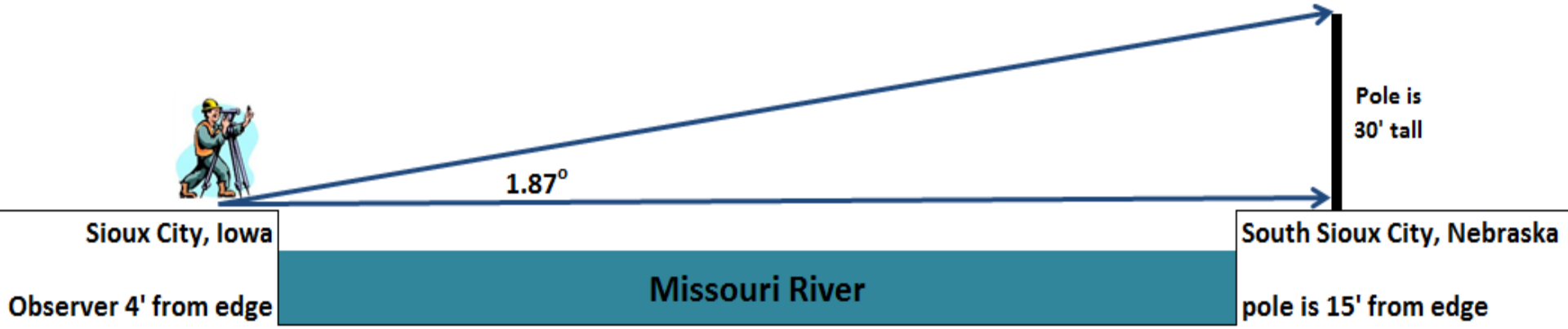

x
20
28°


20
x°
40


2000
50°
x

Real Trig Problems

How wide is the Missouri River?



Real Trig Problems

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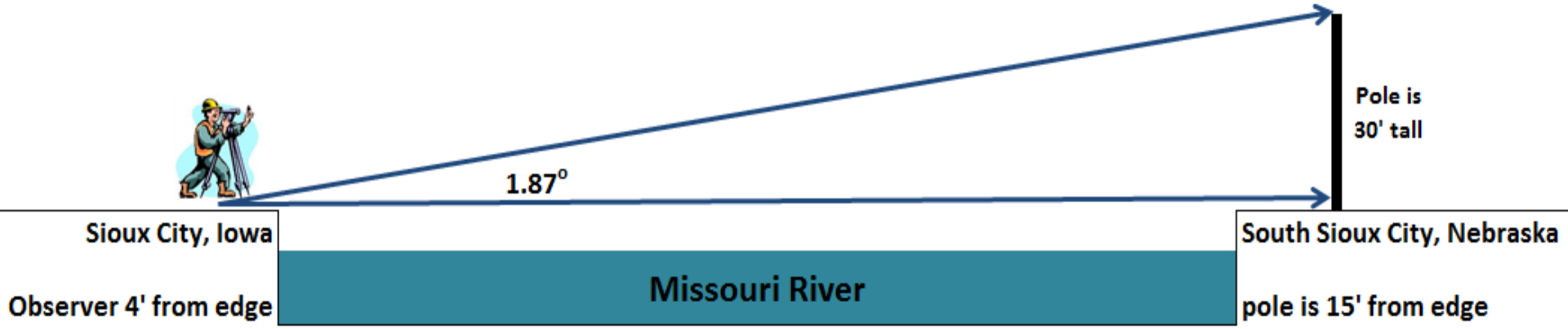
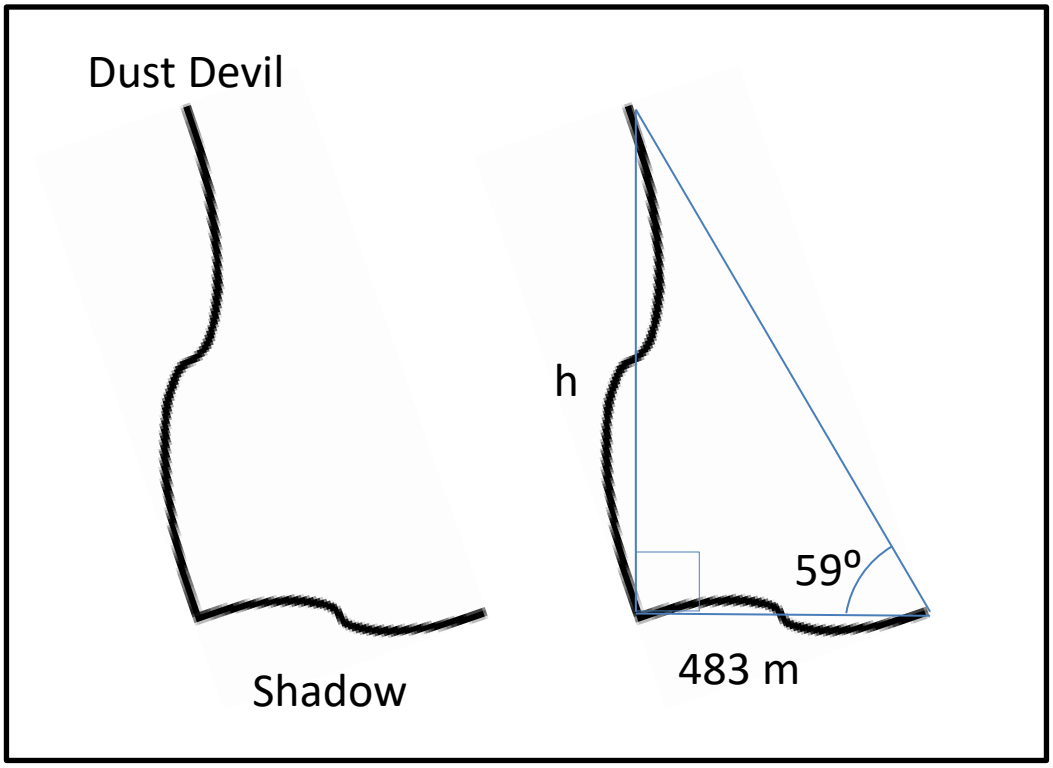
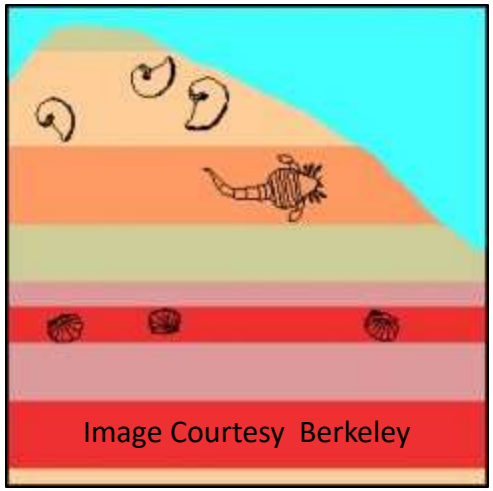


Image Courtesy NASA

- The length of the shadow is approximately 483 meters
- The angle of the Sun over the ground is approximately 59 degrees
- Calculate the height of the dust devil



How Do You Hunt Dinosaurs?



Location 1

Location 2

Location 3

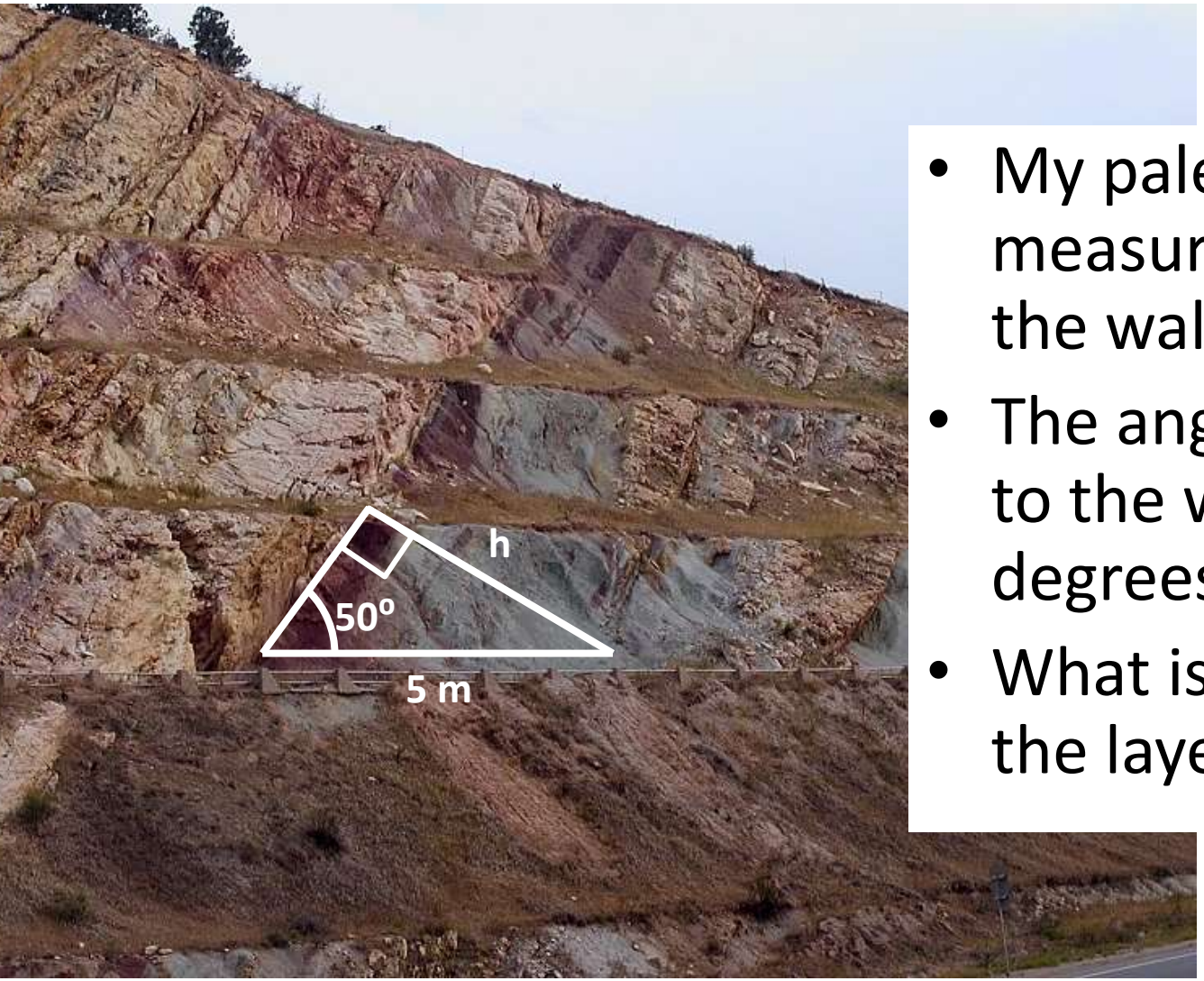
Image Courtesy DMNS

Where can you find ammonites?

- Learn where fossils have been found in the past, and identify the rock layer that had those fossils.
- Trace that layer to new locations and search for new fossils.
- We used trigonometry to measure rock layer thicknesses.

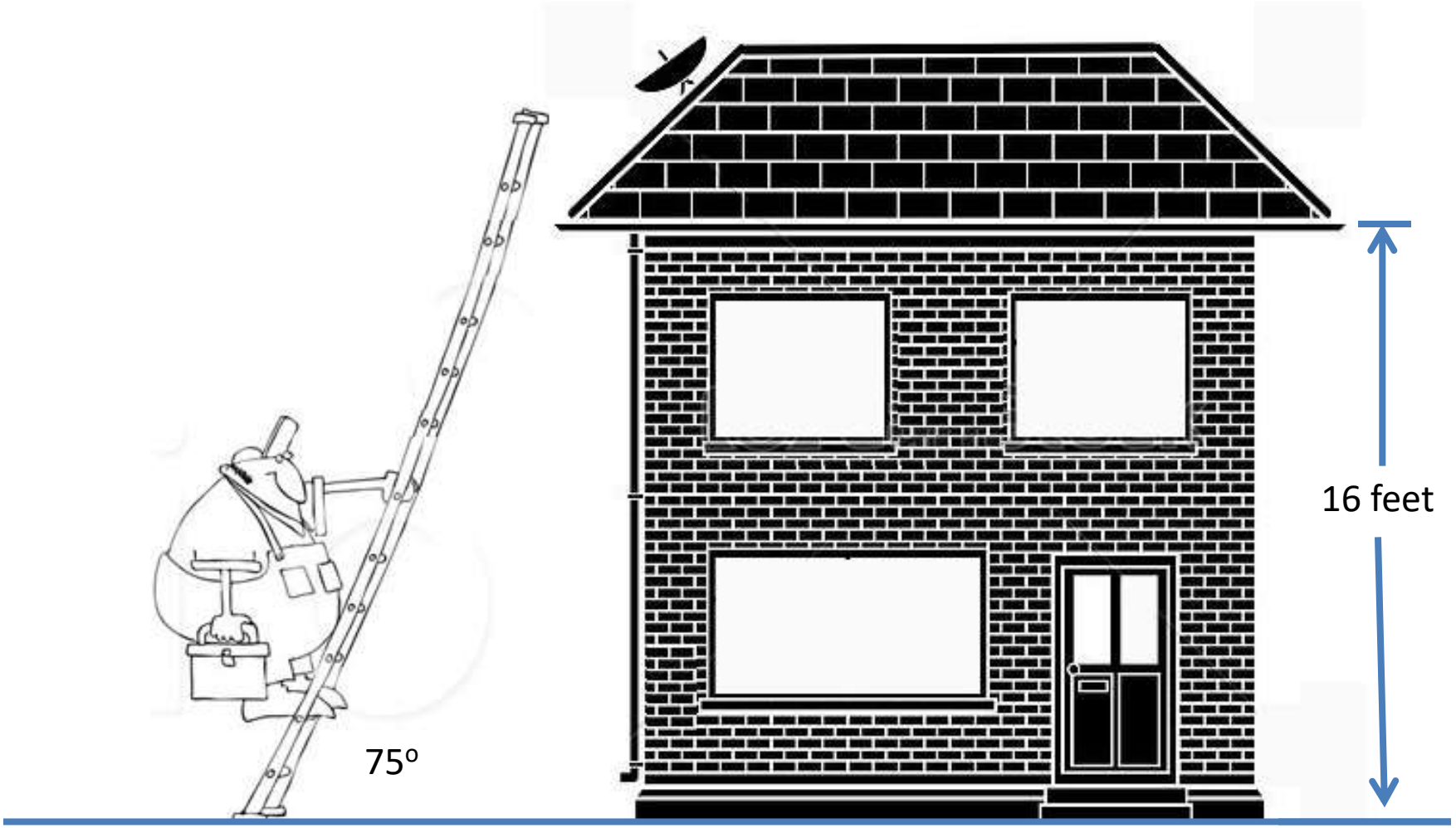
Digging Up Dinosaur Bones

How Thick is This Rock Layer Near Dinosaur Ridge?

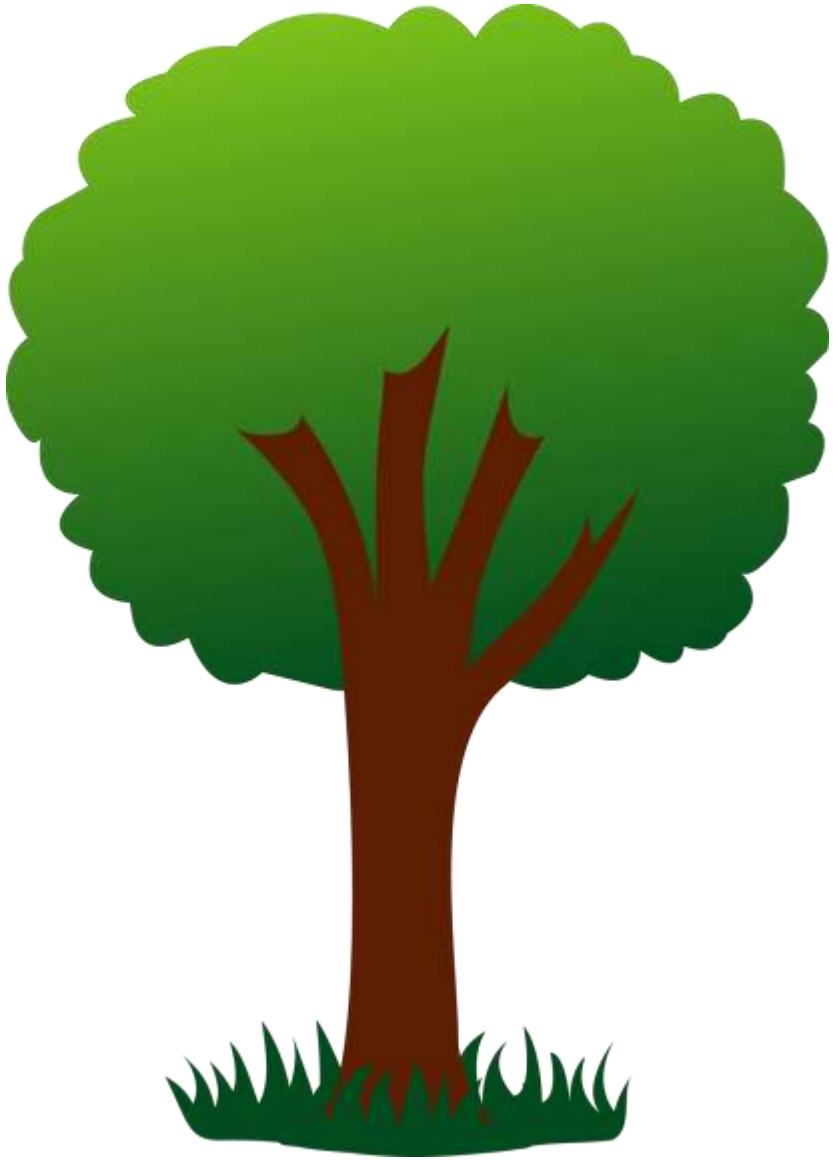


- My paleontology class measured 5 m along the walkway
- The angle of the layer to the walkway was 50 degrees
- What is the height of the layer?

How long should the ladder be?



How tall is the tree?



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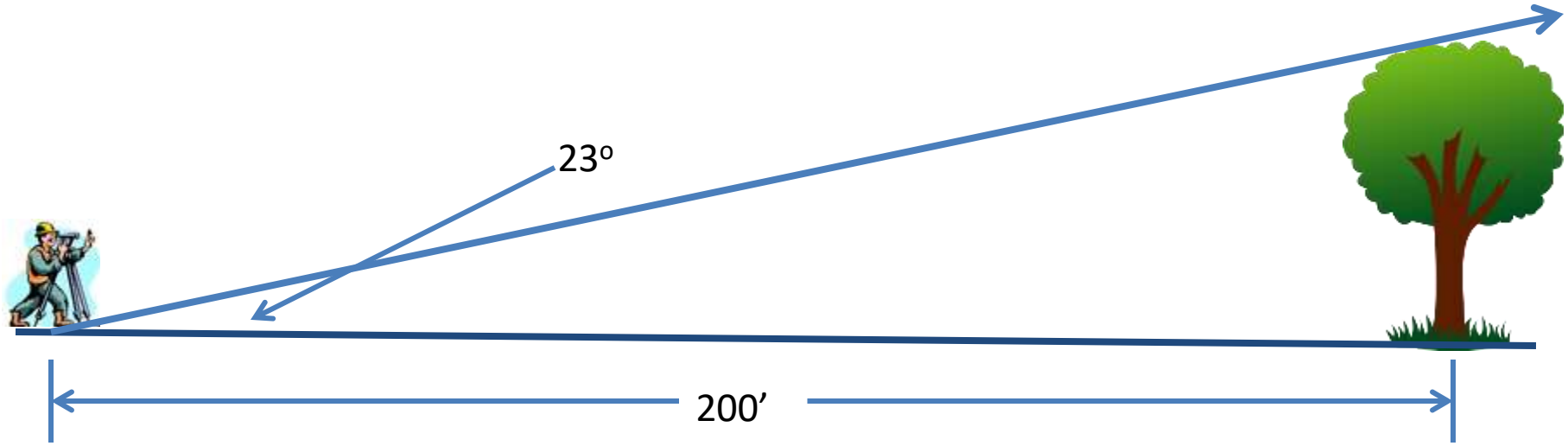
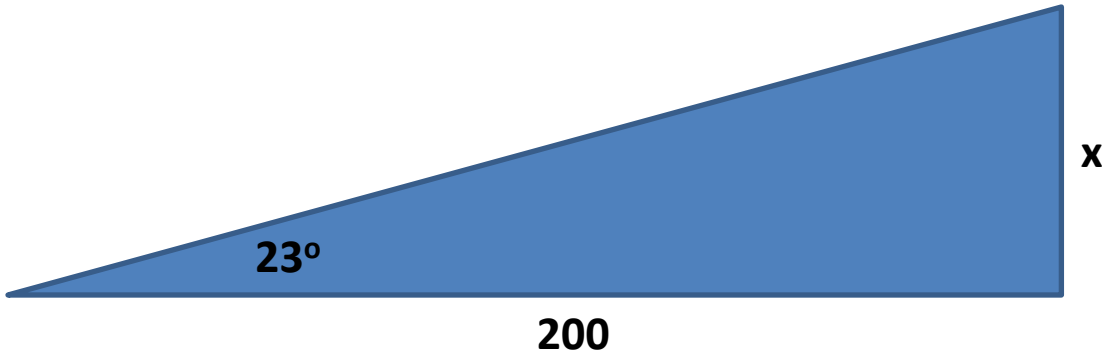




$$\frac{X}{200} = \tan 23^\circ$$

$$X = 200 \tan 23^\circ$$

$$X = 85’$$

Calculus – 3 Areas of Study

- Limits
  - Used to understand undefined values
  - Used to derive derivatives and integrals
- Differential Calculus
  - Uses derivatives to solve problems
  - Great for finding maximums and minimum values
- Integral Calculus
  - Uses integrals to solve problems
  - Great for finding area under a curve
  - Great for finding volumes of 3 dimensional objects

Limits

$$\lim_{x \to 1} \frac{6x-6}{x^2+x-2}$$

$$\lim_{x \to 1} \frac{6\,(x-1)}{(x-1)\,(x+2)}$$

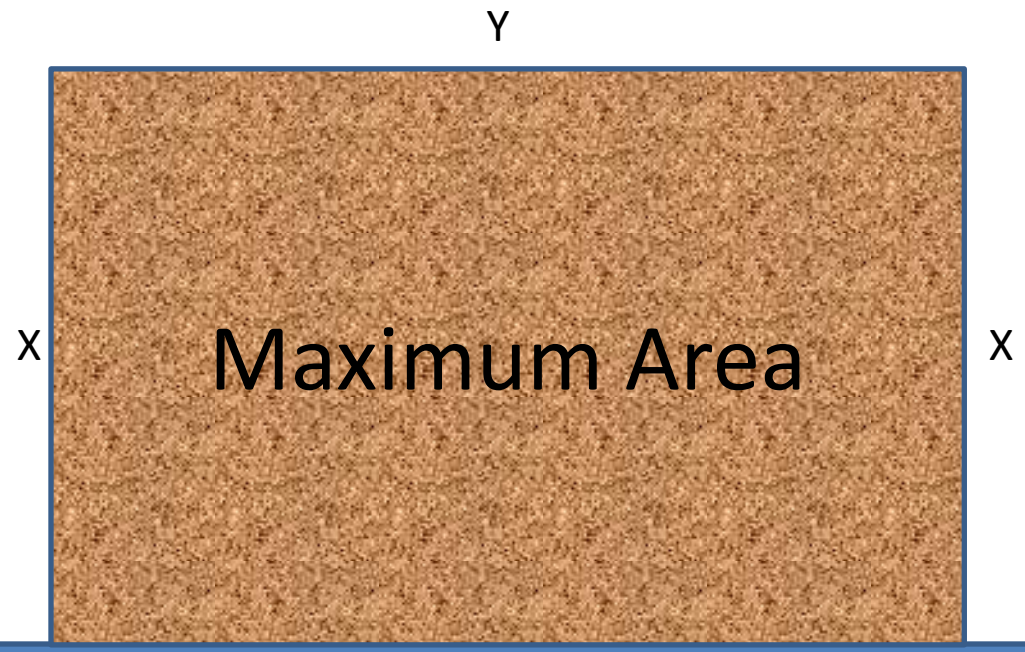
$$\lim_{x \to 1} \frac{6}{(x+2)} = 2$$

Differential Calculus

Function	derivative (slope of tangent line)
$f(x) = x^n$	$f'(x) = nx^{n-1}$

Find the dimensions for max area

- You have 500 feet of fencing
- Build a rectangular enclosure along the river
- Find x and y dimensions such that area is max

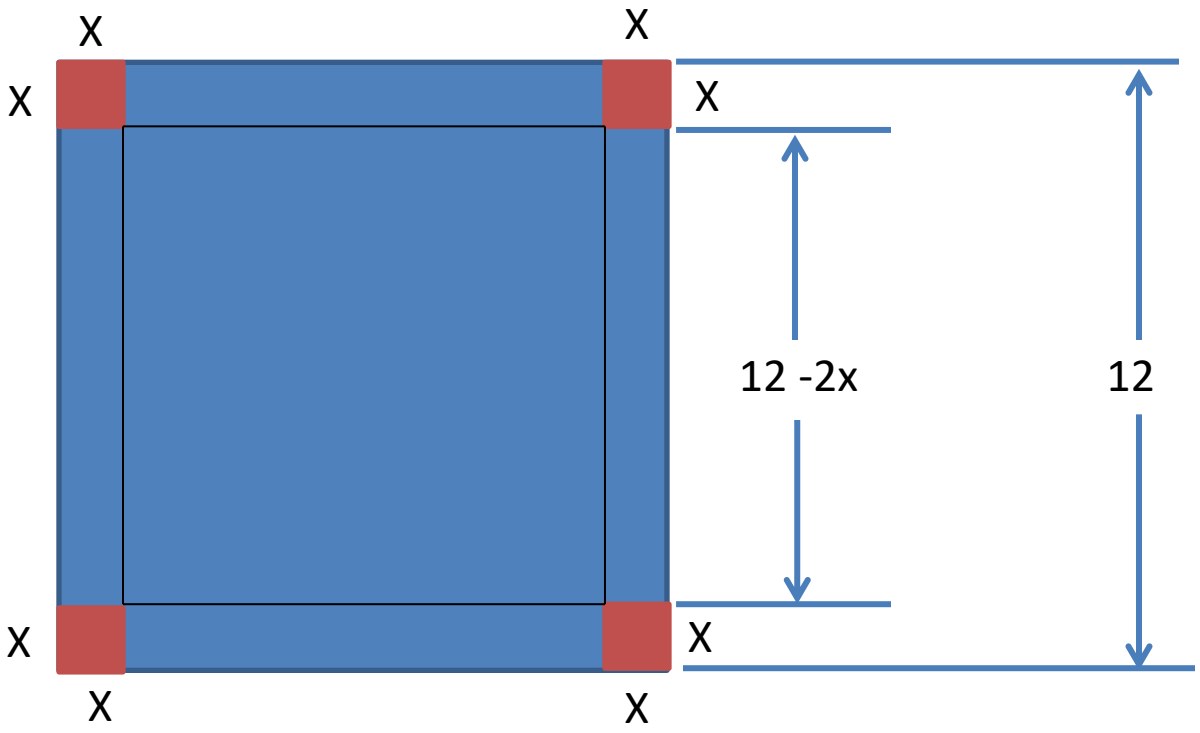


Find the maximum value…

- Using two non-negative numbers
- Whose sum is 9
- The Product of one number and the square of the other number is a maximum

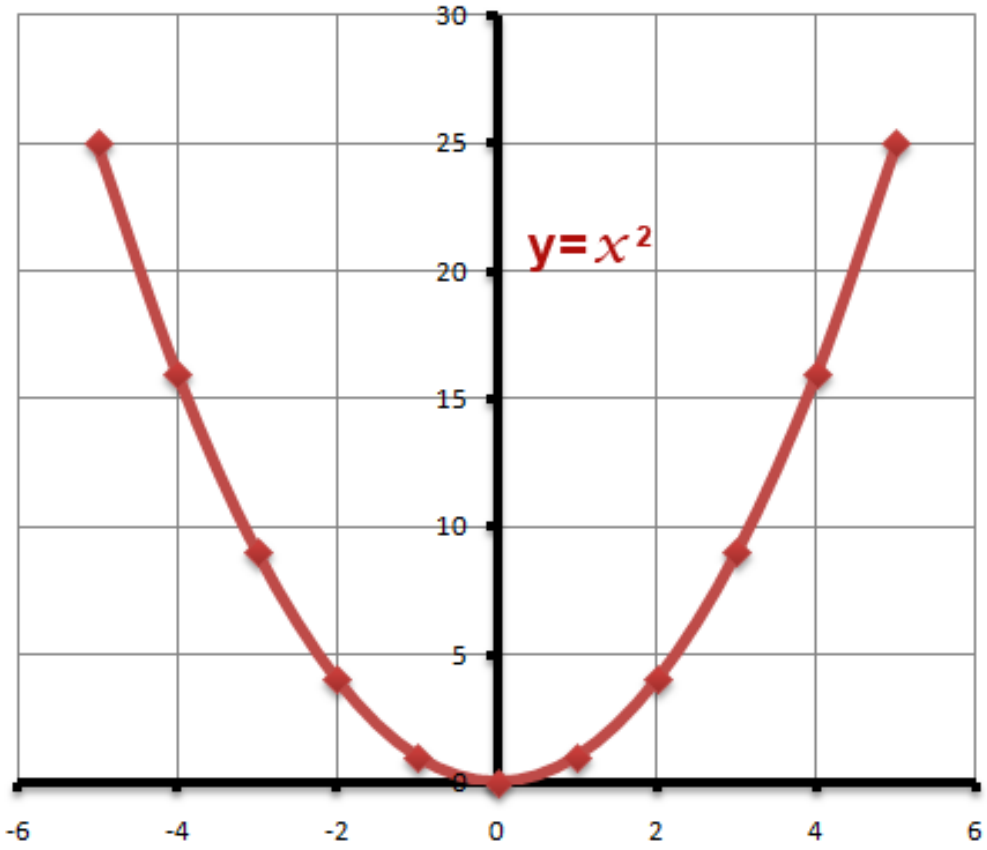
Find dimensions that give max volume…

- One square foot of metal material (12”x12”)
- Cut identical squares out of the four corners
- Fold up sides to made a square pan
- What dimension of x gives the largest volume?



Slope of Tangent Line

- Derivative gives slope of tangent line at point x
- $f(x) = x^2$
- $f'(x) = 2x$
- Point on Curve (1,1)
  - Slope of tangent = 2
- Point on Curve (2,4)
  - Slope of tangent = 4



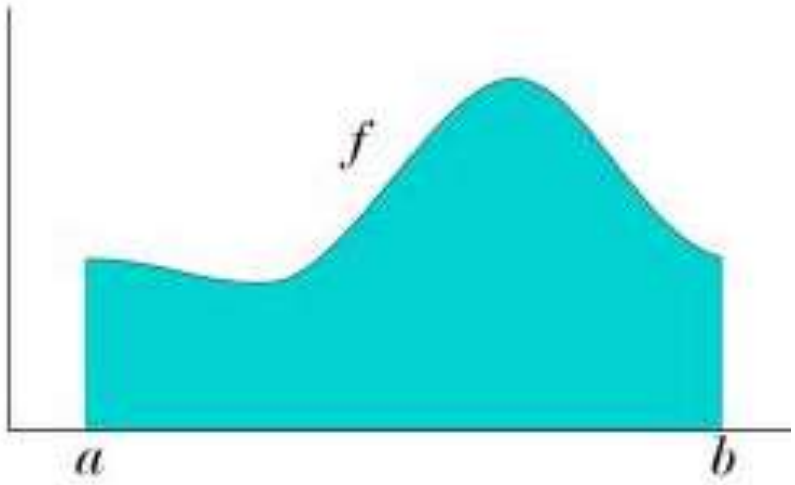
Integral Calculus

Function	Anti-derivative
$f(x) = x^n$	$F(x) = \frac{x^{n+1}}{n+1}$

Integrals

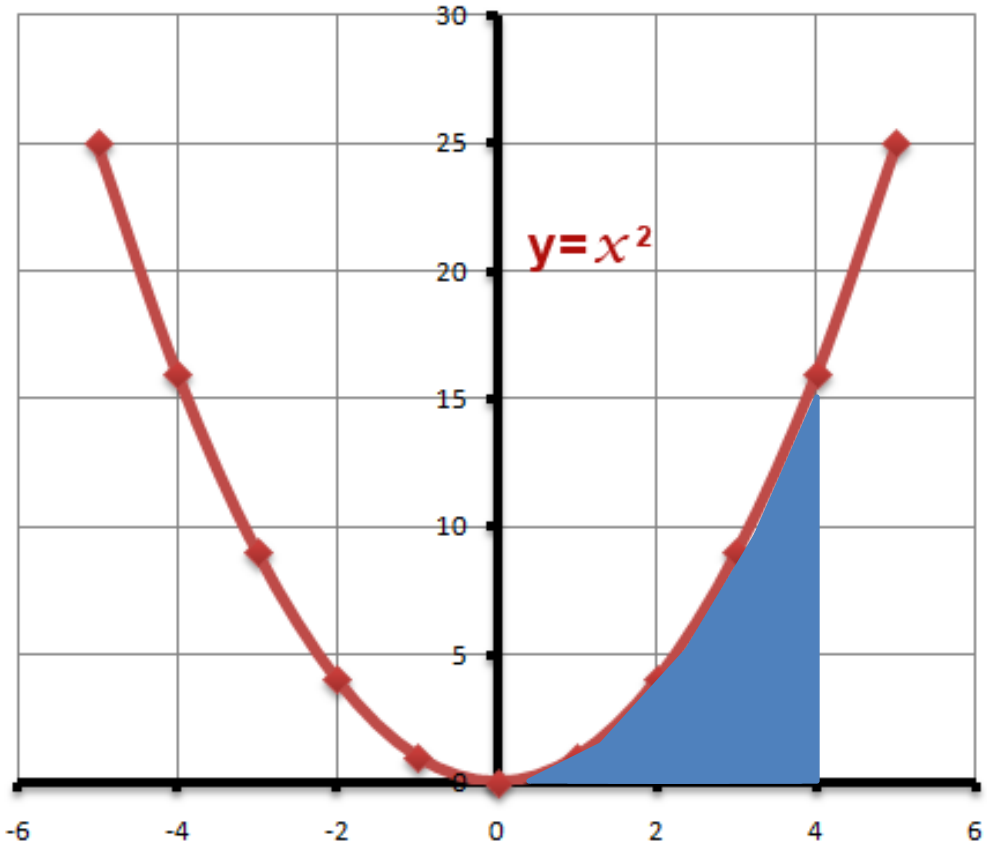
$$\int_a^b f(x)dx = G(b) - G(a)$$

Where G(a) is the anti-derivative of a



Area under a curve

- Integral gives area under the curve
- $f(x) = x^2$
- $\int_0^4 x^2 dx =$



Where can you get Math help?

Math help for Free: http://www.khanacademy.org/